SOME PROBLEMS OF POLARIZATION SENSOR THEORY

Yu. V. Lisitsyn, V. N. Mineev,
and E. Z. Novitskii

We examine the following problems: the charge in a polarization sensor connected across a capacitive load; the relaxation processes in the sensor circuit after shock wave passage through the specimen; the polarization current through a sensor in which two polarization mechanisms exist. The possibility is shown of determining all the unknown parameters of the shock compressed dielectric by means of measurements in the circuits of the short-circuited polarization sensor and a capacitively loaded sensor. Account for the two polarization mechanisms leads to a solution which describes qualitatively several experimental facts. In recent years the polarization of dielectrics in shock waves (hereafter simply wave) has become the object of careful study, which follows three directions: broadening of the class of substances studied, phenomenological description of the experimental results, and clarification of the physical nature of the observed phenomena. The existing phenomenological theories [1-5] relate the current density j in the metering circuit with the bulk resistivity ρ, dielectric permeability ε, specific polarization P_0, polarization decay (mechanical relaxation) time τ, and compression δ of the material behind the shock wave front. In the theories it is assumed that there is a single polarization mechanism and that the dielectric is isotropic. The complexity of the resulting solutions makes it difficult to define uniquely P_0, ε, ρ, and τ without recourse to additional, specially conducted measurements of, for example, ρ and (or) ε. The anomaly in the behavior of the relation $P_0(\delta)$ and the sign change of j in the process of wave propagation through the material (polarity reversal) are not amenable to mathematical description within the framework of the mentioned theories and indicate the possibility of the existence of several polarization mechanisms with different values of P_0 and τ. In the present paper we develop this approach for the case of two mechanisms. Generally speaking, all the solutions examined here can be obtained as a consequence of the Zaidel theory [5] under definite particular assumptions. However, in order to clarify the operations performed it is advisable to use the representation of the polarization sensor as an equivalent electrical circuit [2, 4].

1. Polarization Sensor with Capacitive Load. We shall solve the problem using the same premises as in [4], namely the wave front traveling with the wave velocity D separates the dielectric with initial thickness I_0 into two regions of compressed and noncompressed material (the corresponding characteristics are: ε_2, ρ, δ, u is the mass velocity ε_1, $\rho_1 = \infty$, u = 0); the material behind and ahead of the wave front is isotropic; the dielectric is polarized in the front to the magnitude P_0 with or against the direction of motion of the material; in view of the one-dimensionality of the problem the wavefront is an equipotential surface, as is any other surface parallel to the wave front.

Problem Formulation. We represent the dielectric subjected to shock compression by an electrical circuit at the time t as shown in Fig. 1a. Following [4] we write

$$C_1 = \frac{\alpha_1}{T-t}, \quad C_2 = \frac{\alpha_1 \varkappa}{t}, \quad R = \frac{\rho D t}{\delta} \tag{1.1}$$

(The arguments are made everywhere for unit surface of the shock wave front.)

$$\left(\alpha_1 = \frac{\varepsilon_1}{4\pi D}, \quad \varkappa = \frac{\varepsilon_2 \delta}{\varepsilon_1}, \quad T = \frac{I_0}{D}, \quad \delta = \frac{D}{D-u}\right)$$

Using these expressions, we find the voltage V across the load C_0

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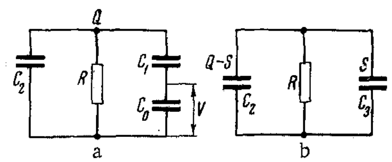

Fig. 1

$$V = \frac{Q}{C_0}\frac{t}{(\varkappa + \gamma)T + (1-\varkappa)t} \qquad \left(\gamma = \frac{C}{C_0},\ C = \alpha_1 \varkappa T^{-1}\right) \tag{1.2}$$

Here Q is the magnitude of the total charge in the system of Fig. 1a, which must be found, C is the capacitance of the completely compressed dielectric. For convenience we transform the circuit of Fig. 1a into the scheme of Fig. 1b. Then

$$C_3 = C_1 C_0 (C_1 + C_0)^{-1}$$

Problem Solution. As shown in Fig. 1b, the magnitude of the charge distributed, between C_2 and C_3, is determined by the following processes: shock polarization, mechanical relaxation (τ), conductivity relaxation ($\theta = \rho\varepsilon_2/4\pi$). The charge change dQ during the time dt can be taken from [4] [Eq. (14)]

$$dQ\,/\,dt = t^{-1}\left[P_0 \exp(-t/\tau) - Q\right] - \theta^{-1}(Q - S) \tag{1.3}$$

Equality of the voltages on the condensors C_2 and C_3 yields

$$S = Q\frac{t}{(\varkappa + \gamma)T + (1-\varkappa)t} \tag{1.4}$$

The initial differential equation (1.3) after substituting (1.4) takes the form

$$\frac{dQ}{dt} + Q\left[\frac{1}{t} + \frac{1}{\theta}\frac{(\varkappa+\gamma)T - \varkappa t}{(\varkappa+\gamma)T + (1-\varkappa)t}\right] = \frac{P_0}{t}\exp\left(-\frac{t}{\tau}\right) \tag{1.5}$$

Its solution with the initial conditions Q = P_0 and t = 0 leads to

$$Q = P_0 \frac{\exp(t/\nu)}{t\left[(\varkappa+\gamma)T + (1-\varkappa)t\right]^{\omega}} \int\limits_0^t \frac{\exp(-t/\mu)}{\left[(\varkappa+\gamma)T + (1-\varkappa)t\right]^{-\omega}}\,dt$$
$$\left(\nu = \frac{1-\varkappa}{\varkappa}\theta,\ \mu = \frac{\tau\theta(1-\varkappa)}{\theta + \varkappa(\tau - \theta)},\ \omega = \frac{T}{\theta}\frac{\varkappa+\gamma}{(1-\varkappa)^2}\right) \tag{1.6}$$

By the simple algebraic operations used in [4] we can show that the system charge in the short-circuited polarization sensor circuit is

$$Q = P_0 \frac{\exp(t/\nu)}{t\left[\varkappa T + (1-\varkappa)t\right]^{\varphi}} \int\limits_0^t \frac{\exp(-t/\mu)\,dt}{\left[\varkappa T + (1-\varkappa)t\right]^{-\varphi}},\quad \varphi = \frac{T\varkappa}{\theta(1-\varkappa)^2} \tag{1.7}$$

This expression is a particular case of (1.6) for $C_0 \gg C$ ($\gamma \gg 1$, $\varkappa \approx 1$). If the conductivity of the material behind the wave front is small, the quantity Q is the system bound charge Q_∞. It can be obtained by solving the differential equation (1.5) with $\theta \gg T$

$$Q_\infty = \frac{P_0\tau}{t}\left[1 - \exp(-t/\tau)\right] \tag{1.8}$$

Substituting (1.6) into (1.2) we obtain in the general case

$$V = \frac{P_0}{C_0}\frac{\exp(t/\nu)}{\left[(\varkappa+\gamma)T + (1-\varkappa)t\right]^{\omega+1}} \int\limits_0^t \frac{\exp(-t/\mu)}{\left[(\varkappa+\gamma)T + (1-\varkappa)t\right]^{-\omega}}\,dt \tag{1.9}$$

and in the absence of conductivity behind the wave front

$$V = \frac{P_0\tau}{C_0}\frac{1 - \exp(-t/\tau)}{(\varkappa+\gamma)T + (1-\varkappa)t} \tag{1.10}$$

2. Relaxation Processes in Polarization Sensor Circuit. Let us find the magnitude of the charge in the system of Fig. 1b for those cases in which the wave leaves the test material at the metering electrode without reflection. We shall examine times $t \geq T$. We denote $t' = t - T$ and $Q_{t=T} = Q^\circ$. Obviously, Q° equals Q for $t' = 0$.

All the parameters of the circuit of Fig. 1b are constants. Thus

$$C_2 = C = \alpha_1\varkappa T^{-1}, \qquad R = \rho I_0 \delta^{-1}, \qquad C_3 = C_0$$

Only the charge Q remains dependent on the time t', and this charge will decay as a result of the existence of the relaxational processes, and only a part of the total charge, the bound charge Q_∞, will decay

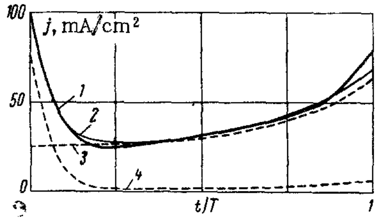

Fig. 2

with time τ, while the entire system charge Q decays with the time θ.

The magnitude of the bound charge at the time $t' = 0$ (Q°_{∞}) is found from (1.8)

$$Q_{\infty}{}^{\circ} = \tau P_0 T^{-1} [1 - \exp(-T / \tau)] \tag{2.1}$$

Then Q_{∞} for $t' > 0$ is

$$Q_{\infty} = Q_{\infty}{}^{\circ} \exp(-t' / \tau)$$

and the corresponding decrease of Q_{∞} is

$$dQ_1 = -Q_{\infty}{}^{\circ} \tau^{-1} \exp(-t' / \tau)\, dt' \tag{2.2}$$

The charge decrease as a result of conductivity of the material behind the wave front is

$$dQ_2 = -\frac{V}{R}\, dt' = -\frac{Q dt'}{R(C + C_0)} = -\frac{\gamma Q dt'}{\theta(1+\gamma)} \tag{2.3}$$

Summing (2.2) and (2.3), we obtain the differential equation

$$\frac{dQ}{dt'} + Q\frac{\gamma}{\theta(1+\gamma)} = -Q_{\infty}{}^{\circ}\tau^{-1}\exp(-t'/\tau) \tag{2.4}$$

The solution of (2.4) with the initial conditions

$$Q^{\circ} = P_0 T^{-1} \frac{\exp(T/\nu)}{[T(1+\gamma)]^{\omega}} \int_0^T \frac{\exp(-t/\mu)\, dt}{[(\varkappa+\gamma)T + (1-\varkappa)t]^{-\omega}} \tag{2.5}$$

for $t' = 0$ yields

$$Q = Q_{\infty}{}^{\circ} \frac{m}{\tau - m} \exp\frac{-t'}{m} \left\{1 - \exp\left[t'\left(\frac{1}{m} - \frac{1}{\tau}\right)\right] + \frac{Q^{\circ}}{Q_{\infty}{}^{\circ}} \frac{\tau - m}{m}\right\} \qquad \left(m = \frac{\theta(C + C_0)}{C}\right) \tag{2.6}$$

Then

$$V = Q(C + C_0)^{-1}$$

Let us analyze this expression briefly. For $C_0 \to \infty$ (which corresponds to the short-circuited circuit condition) and nonzero θ, (2.6) implies

$$Q = Q^{\circ} - Q_{\infty}{}^{\circ} [1 - \exp(-t' / \tau)] \tag{2.7}$$

The derivative of (2.7) with respect to t' yields the value of the relaxation current

$$j_+ = -Q_{\infty}{}^{\circ} \tau^{-1} \exp(-t' / \tau),$$

or after substitution of Q°_{∞} from (2.1)

$$j_+ = -P_0 T^{-1} [1 - \exp(-T / \tau)] \exp(-t' / \tau), \tag{2.8}$$

which agrees with the solution of [6], obtained within the framework of the Allison theory [1], which does not take into account the conductivity of the material behind the wave front; and this means that the current decay in the short-circuited polarization sensor circuit is determined only by the relaxation time τ.

The initial current j_0 appearing at the moment the shock wave enters the test specimen is independent of the relaxation processes [4] and is determined by the expression $j_0 = P_0(\varkappa T)^{-1}$. For known $k = j_+/j_0$ we find the value of $\varkappa$

$$\varkappa = k\{[\exp(-T / \tau) - 1] \exp(-t' / \tau)\}^{-1} \tag{2.9}$$

If τ and $\varkappa$ are found using (2.8) and (2.9) from experiments in the short-circuited circuit, then it is easy to find the values of

$$P_0 = j_0 \varkappa T, \qquad C = \alpha_1 \varkappa T^{-1}$$

and Q°_{∞} from (2.1). The voltage in the scheme of Fig. 1b for $t' = 0$ ($t = T$) determined the charge $Q^{\circ} = V(C + C_0)$, expressed by (2.5). The voltage in this same circuit for any $t' > 0$ makes it possible to find from (2.6) the one remaining unknown m. The parameters $\varkappa$ and m contain in themselves the unknowns ε_2 and ρ.

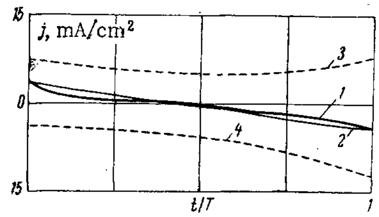

Fig. 3

3. Polarization Current in Sensor in Presence of Two Polarization Mechanisms. The fact of anomalous polarization in ionic crystals, and also the change in certain cases of the sign of the polarization current in the process of wave passage through the specimen, can be explained by the existence of at least two independent polarization processes with different signs. Each of these processes must be characterized by its own values of τ and P_0.

Let us examine the short-circuited case. For definiteness we assume that $P_0^{(1)}$ is always positive, while $P_0^{(2)}$ can take both positive and negative values. In the following arguments we follow [4].

Charge increase in the system takes place only in C_2 (Fig. 1a) as a result of polarization of additional dielectric layers

$$dQ_1 = (P_0^{(1)} \pm P_0^{(2)} - Q)\, t^{-1}\, dt \tag{3.1}$$

The charge reduction dQ_2 owing to mechanical relaxation is expressed as

$$dQ_2 = -t^{-1} \quad \{P_0^{(1)}\,[1 - \exp(-t/\tau_1)] \pm P_0^{(2)}\,[1 - \exp(-t/\tau_2)]\}dt \tag{3.2}$$

and the charge decrease owing to conductivity is

$$dQ_3 = -\,(Q - S)\,\theta^{-1}\,dt \tag{3.3}$$

Here S is the charge flowing in the short-circuited sensor circuit, whose magnitude must satisfy the voltage equality condition

$$(Q - S)\, C_2 = SC_3$$

Summing (3.1), (3.2), (3.3), we find

$$\frac{dQ}{dt} = \frac{1}{t}\left\{P_0^{(1)} \exp\frac{-t}{\tau_1} \pm P_0^{(2)} \exp\frac{-t}{\tau_2} - Q\right\} - \frac{Q-S}{\theta}\, dt \tag{3.4}$$

or in terms of the charge S

$$\frac{dS}{dt} + S\left[\frac{1-\varkappa}{\varkappa T + (1-\varkappa)\, t} + \frac{1}{\theta}\,\frac{\varkappa (T-t)}{\varkappa T + (1-\varkappa)\, t}\right] = \frac{P_0^{(1)} \exp(-t/\tau_1) \pm P_0^{(2)} \exp(-t/\tau_2)}{\varkappa T + (1-\varkappa)\, t} \tag{3.5}$$

The solution (3.5) with zero initial conditions makes it possible to write the time dependence of the polarization current density

$$\begin{aligned} j = {} & \frac{P_0^{(1)} \exp(-t/\tau_1)}{\varkappa T + (1-\varkappa)\, t}\left\{1 - \frac{\theta^{-1}\varkappa (T-t) + (1-\varkappa)}{[\varkappa T + (1-\varkappa)\, t]^{\varphi}} \exp(t/\mu_1)\int_0^t \frac{[\varkappa T + (1-\varkappa)\, t]^{\varphi-1}}{\exp(t/\mu_1)}\, dt\right\} \pm \frac{P_0^{(2)} \exp(-t/\tau_2)}{\varkappa T + (1-\varkappa)\, t} \\ & \times \left\{1 - \frac{\varkappa (T-t)\,\theta^{-1} + (1-\varkappa)}{[\varkappa T + (1-\varkappa)\, t]^{\varphi}} \exp(t/\mu_2)\int_0^t \frac{[\varkappa T + (1-\varkappa)\, t]^{\varphi-1}}{\exp(t/\mu_2)}\, dt\right\} \end{aligned} \tag{3.6}$$

which is in essence the superposition of two solutions [4].

Continuing the arguments, we find that the relaxation current is also the superposition of two solutions (2.8), i.e.,

$$j_+ = -\left\{\frac{P_0^{(1)}}{T}\left[1 - \exp\frac{-T}{\tau_1}\right]\exp\frac{-t'}{\tau_1} \pm \frac{P_0^{(2)}}{T}\left(1 - \exp\frac{-T}{\tau_2}\right)\exp\frac{-t'}{\tau_2}\right\}$$

All this makes it possible to evaluate in a new light both the results of τ measurements over a wide range of pressures and the difficulties which arise in the phenomenological description of the experimental curves of J(t) with the aid of the existing theories.

Figure 2 shows for comparison: the experimental curve 1 obtained in an experiment with NaCl with a pressure of 100 kbar behind the wave front, T = 0.825 sec; the theoretical curve 2, obtained by superposing curve 2 ($P_0 = 4.12 \cdot 10^{-8}$ C/cm², $\varkappa = 2$, $\tau = \theta = 1.65$ μsec) and curve 4 ($P_0 = 1.24 \cdot 10^{-7}$ C/cm², $\varkappa = 2$, $\tau = 0.04$ μsec, $\theta = 1.65$ μsec).

Figure 3 shows: the experimental curve 1 obtained in experiments with KBr with a pressure of 78 kbar at the wavefront, $T = 1.31$ μsec, the calculated curve 2 obtained by superposing curve 3 (($P_0 = 1.97 \cdot 10^{-8}$ C/cm^2, $\varkappa = 2$, $\tau = 0.65$ μsec, $\theta = 6.5$ μsec) and curve 4 ($P_0 = -0.98 \cdot 10^{-8}$ C/cm^2, $\varkappa = 2$, $\tau = \theta = 6.5$ μsec).

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